Dynamic Force Analysis of Julius Cheezer on Rat Legs

Brandon Lim | 2/29/2024 | Draft 1

## **Executive Summary**

A dynamic force analysis of vehicle motion along the connecting track Julius Cheezer revealed a resultant force of 17577.9 newtons. This resultant force from the vehicles motion causes normal and shear stresses in the A-Frame supports that hold the connecting track. The normal and shear stresses are 1.26 MPa and 0.02 MPa respectively while the maximum normal and shear stress of the A36 steel used in the A-Frame are 550 MPa and 80000 MPa respectively. This analysis ensures the safety of the A-Frame supports when under a dynamic force of the vehicle on Julius Cheezer.

## **Introduction**

The University of Utah has a brand-new roller coaster with exciting thrills and sections that have been expertly engineered. This new rollercoaster has been themed after a rat uprising and is designed to support its riders effectively and safely. This coaster consists of three main thrills, one main climb, one vehicle, four connecting tracks, and many supports that frame the whole ride. Julius Cheezer is a section of the track that includes a wide 360-degree banked turn around a 20-meter radius as seen in Figure 1-1.

A black and white drawing of a circle

Description automatically generated

***Figure 1-1:*** *Aerial view of the Julius Cheezer connecting track. The roller coaster vehicle will approach from the left when looking from the ariel view and go through a 360-degree banked turn around a 20-meter radius.*

The frame that supports Julius Cheezer is an A-Frame design that consists of tubular legs ranging from 3 meters to 100 meters off the ground. The A-Frame connects to Julius Cheezer by either holding the track adjacently or in between its legs as seen in Figure 1-2. To ensure rider safety around Julius Cheezer, a dynamic force analysis around this section was performed and the resulting forces were compared to the A-Frame’s specifications.

A black and white drawing of a triangle with a ball

Description automatically generated

***Figure 1-2:*** *Front view of the A-Frame supports holding a section of the track. The two pillars come together creating the “A-Frame” and has a section extending down that attaches to the track.*

## **Kinematics and Physics of Julius Cheezer**

As seen in Figure 1-1, Julius Cheezer presents unique challenges to the physics of its velocity and acceleration due to its nature being primarily within the realm of angular motion. Riders will enter the connecting track before the 360-degree banked turn at approximately 30 meters per second. After entering the 20-meter radius banked turn, the vehicle and its riders will complete the entire wrap-around within 10 seconds. Travelling around the banked turn will result in accelerations and corresponding forces that are much more complex than simple linear motion. Analysis of the dynamic forces experienced by riders and the vehicle during Julius Cheezer is broken down by centripetal force analysis.

A diagram of a circle with arrows and lines

Description automatically generated

***Figure 2-1:*** *Breakdown of centripetal acceleration around a curve where R is the radius of the curve, is the acceleration of the body around the curve in the tangential direction, and is the acceleration of the body around the curve in the normal direction.*

As seen in Figure 2-1, when a body is in motion around a circular or curved path, the body will experience two accelerations, an acceleration in the tangential direction and an acceleration in the normal direction. The acceleration in the tangential direction is responsible for the vehicle’s motion linearly and allows it to translate forward from the reference point of a passenger. The acceleration in the tangential direction is given by

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

where t is the time it takes to travel around the curve (seconds), V is the velocity of the vehicle around the wrap-around (meters/second), l is the length of the wrap-around (meters), and are the first derivative of velocity and second derivative of length respectively (meters/). The acceleration in the normal direction is responsible for the translation of the vehicle along the curve and is perpendicular to the tangential acceleration. The acceleration in the normal direction is given by

|  |  |  |
| --- | --- | --- |
|  |  | (2) |
|  |  |  |

where t is the time it takes to travel around the curve (seconds), V is the velocity of the vehicle around the wrap-around (meters/second), l is the length of the wrap-around (meters), R is the radius of the curve (meters), and is the first derivative of length.

## **Resulting Forces from Vehicle Motion on Julius Cheezer**

Using Newtons second law, the accelerations caused by the motion around the curved connecting track, Julius Cheezer, can be transformed into forces.

A diagram of a circle with lines and points

Description automatically generated

***Figure 3-1:*** *Breakdown of centripetal forces around a curve where R is the radius of the curve, is the force of the body around the curve in the tangential direction, is the force of the body around the curve in the normal direction, and F is the resultant force from* .

Figure 3-1 is a transformation of Figure 2-1 where the acceleration in the tangential and normal directions became the forces in the tangential and normal directions respectively. This was done by utilizing Newtons second law which is given by

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

where m is the mass of vehicle with its passengers (kilograms), and a is the acceleration of the vehicle around Julius Cheezer (meters/). When combining equations (1) and (2) with equation (3), the force transformation is given by

|  |  |  |
| --- | --- | --- |
|  |  | (4) |
|  |  | (5) |
|  |  |  |

where is the mass of vehicle with the passengers (kilograms), l is the length of the wrap-around (meters), t is the time it takes to go around the wrap-around, and R is the radius of the wrap-around. Using the forces calculated in the tangential and normal directions, a resultant force can be found that is indicated in Figure 3-1 by the red vector. The magnitude of the resultant force can be calculated using the equation

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

where is the force in the tangential direction given by equation (4) (Newtons), and is the force in the normal direction given by equation (5) (Newtons).

## **Stresses acting on A-Frame Supports from Dynamic Forces**

The dynamic force analysis of the vehicle along Julius Cheezer creates stresses in the A-Frame supports. Due to the orientation of the forces being horizontal and parallel to the ground, the resultant force, calculated using equation (6), creates normal bending stresses and transverse shear stresses in the A-Frames legs. The maximum transverse shear stress and maximum normal bending stress that result from the resultant force is located at the base of the A-Frame next to ground. The maximum normal bending stress is given by the equation

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

where c is the radius of the A-frame leg (meters), M is the maximum bending moment (Newton-meter), and I is the moment of inertia of the A-frame leg (. The maximum bending moment can be given by

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

where F is the resultant force (Newtons), and d is the vertical distance from the ground to where the track connects to the A-frame (meters). The moment of inertia is given by

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

where d is the diameter of the A-Frame leg (meters). After substituting equations (8) and (9) into equation (7), the maximum normal bending stress is calculated. The transverse shear stress is given by the equation

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

where f is the resultant force (Newtons), and A is the cross-sectional area of the cylindrical A-Frame support leg.

## **Results of Force and Stress Analysis**

**Table 1.1:** Dynamic Force Analysis Results

|  |  |  |
| --- | --- | --- |
| Parameter | Symbol | Value |
| Acceleration Tangential |  | 1.26 m/s |
| Acceleration Normal |  | 2.89 m/s |
| Force Tangential |  | 2772 N |
| Force Normal |  | 17358 N |
| Resultant Force | F | 17577.9 N |

**Table 1.2:** Stress Analysis Results

|  |  |  |
| --- | --- | --- |
| Parameter | Symbol | Value |
| Normal Bending Stress |  | 1.12 MPa |
| Transverse Shear Stress |  | 0.02 MPa |

**Table 1.3:** Material Properties of A-Frame A-36 Steel

|  |  |  |
| --- | --- | --- |
| Parameter | Symbol | Value |
| Maximum Normal Stress |  | 550 MPa |
| Transverse Shear Stress |  | 80000 MPa |

## **Conclusion**

A dynamic force analysis of Rat Legs was performed to ensure the safety of University of Utah Students along the Ute Coaster’s connecting track Julius Cheezer. As seen in Table 1.1, a dynamic force analysis of the vehicle traveling along Julius Cheezer revealed a resultant force of 17577.9 newtons created by the normal and tangential accelerations alongside Newtons second law. This resultant force was used in a stress analysis to calculate normal bending stress and transverse shear stress in the A-Frame support legs. The stresses experienced by the A-Frames that are generated by the dynamic forces of the vehicle along Julius Cheezer are small in comparison to the maximum stresses of the A36 steel used as the A-Frame legs. As seen in Table 1.2, the normal bending stress and transverse shear stress were 1.12 MPa and 0.02 MPa respectively. In Table 1.3, the maximum normal and maximum shear stress that the A36 steel can handle is 550 MPa and 80000 MPa respectively. This results in a safety factor of 491 for normal stress and 4000000 for the shear stress. In conclusion, the dynamic force analysis of Rat Legs reveals that the A-Frame can adequately and safely support the dynamic forces generated by Julius Cheezer.

*.*